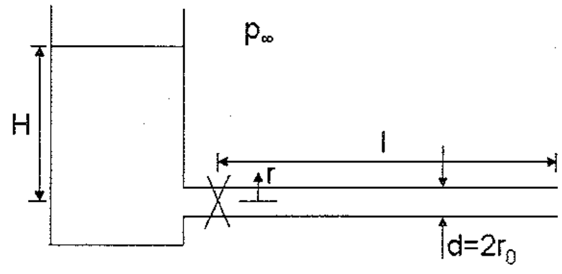
TTY / Energia- ja prosessitekniikan laitos

ENER-4010 Virtausoppi

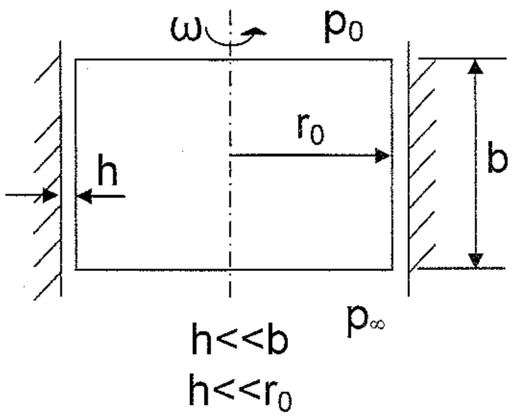
Tentti 28.11.2008

Sallittu kirjallisuus: kaavakokoelma.
Palauta jaettu kaavakokoelma tentin jälkeen.
Älä tee kaavakokoelmaan merkintöjä.

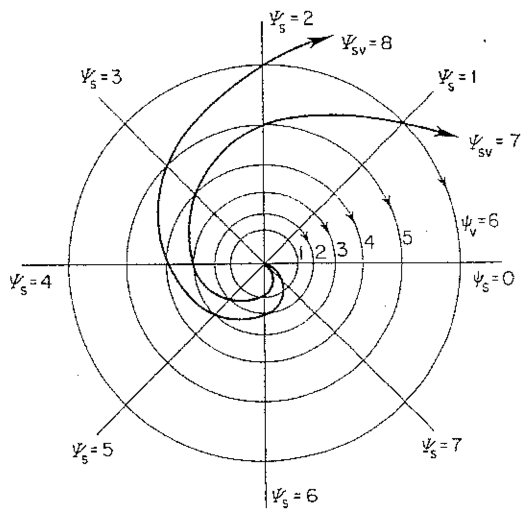
1. Säiliössä pinnankorkeus pidetään vakiona H. Tietyllä ajanhetkellä venttiili avataan täysin, jolloin virtaus alkaa (l>>d). Putki on täynnä nestettä.
   a. Mikä on nopeutta u(r,t) hallitseva osittaisdifferentiaaliyhtälö reuna- ja alkuehtoineen? Virtaus on <u>laminaari</u>.
   b. Mikä on nopeusprofiili u(r), kun nopeus on saavuttanut lopullisen arvon?
   c. Mikä on stationääri tilavuusvirta?
   d. Kerro miten laskisit edellä olevan tehtävän, jos virtaus on turbulentti? Oleta kitkakerroin tunnetuksi?



2. Kuvassa näkyvä sisempi sylinteri pyörii kulmanopeudella ω. Raon, leveys h, yli on paine-ero $\Delta p = p_0 - p_\infty$. Oleta, että <u>laminaari</u> virtaus raossa voidaan laskea kahden levyn välissä tapahtuvana virtauksena karteesisessa koordinaatistossa ja että pyörimissuunnassa ja aksiaalisuunnassa virtaukset voidaan käsitellä erikseen ilman, että ne vaikuttavat toisiinsa.
   a. Mikä on vuotovirtaus?
   b. Mikä on pyörimiseen tarvittava teho?
   c. Mikä on laitteen käyttämiseen tarvittava teho? Huomaa, että vuotovirtauksen pumppaukseen tarvitaan tehoa.
   d. Pitääkö tehtävässä esitetty oletus paikkaansa, ts. kulman suuntainen ja aksiaalisuuntainen nopeus voidaan käsitellä erikseen? Perustele.

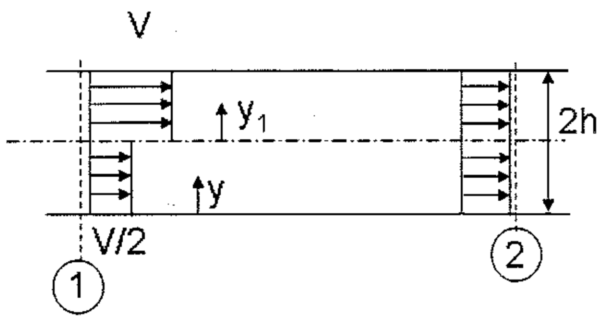


3. Kuva esittää potentiaalivirtausta, joka syntyy kun nielu (tuotto m) ja kiertovirtaus (sirkulaatio Γ) yhdistetään.
   a. Mikä on syntyneen virtauksen kompleksinen nopeuspotentiaali?
   b. Mikä on virtafunktio?
   c. Mikä on m ja Γ, jos resultanttinopeus kohdassa r = 0,2 m on 30 m/s?
   d. Mikä on paine kohdassa r = 0,2 m, jos se kohdassa r = 0,1 m on 1 bar?

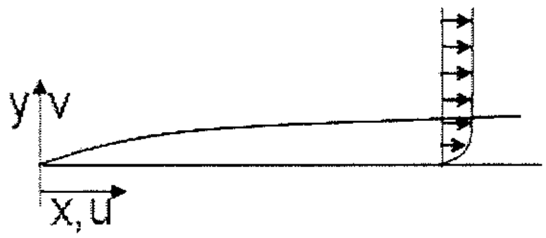


Käännä paperi →

4. Kaksi virtausta, joissa on tasainen nopeusjakauma, sekoittuu levyjen välissä.
   a. Soveltamalla kuvan kontrollipintaan jatkuvuus- ja liikeyhtälöitä laske paine leikkauksessa 2, jos se leikkauksessa 1 on p_0. Jätä seinämäkitka huomioonottamatta.
   b. Sama kuin a-kohta, mutta nyt nopeus leikkauksessa 2 ei ole tasainen, vaan noudattaa täysin kehittyneen turbulentin virtauksen jakaumaa $\frac{u}{u_m} = \left(\frac{y}{h}\right)^{1/7}$.
   c. Sama kuin b-kohta, mutta lopullinen laminaari profiili on $u = \frac{3}{2}V\left[1-\left(\frac{y_1}{h}\right)^2\right]$.



5. Tasolevyn ohi virtaa ilmaa nopeudella 10 m/s. $\nu = 1{,}6 \cdot 10^{-5}$ m²/s, $\rho = 1{,}2$ kg/m³.
   a. Laske mitkä ovat nopeudet u ja v kohdassa x = 1 m ja y = 1 mm, jos rajakerros on laminaari.
   b. Laske nopeudet u ja v, samassa kohdassa, jos virtaus on turbulentti levyn alusta lähtien.



25200 VIRTAUSOPPI (tentissä sallittu kirjallisuus)

VIRTAUSTA HALLITSEVAT YHTÄLÖT

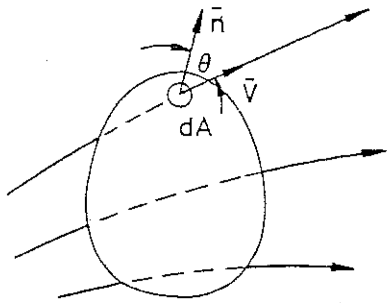
P = mp

$$\frac{dP}{dt} = \frac{\partial}{\partial t}\int_V \rho\, p dV + \int_A \rho\, p\overline{V} \cdot \overline{n} dA$$

1. **Jatkuvuusyhtälö**

P = m, p = 1

$$\frac{dm}{dt} = 0 \qquad \frac{\partial}{\partial t}\int_V \rho\, dV + \int_A \rho \overline{V} \cdot \overline{n} dA = 0$$



2. **Liikeyhtälö**

$P = m\overline{V}$, $p = \overline{V}$

$$\sum \overline{F} = \frac{d}{dt}\left(m\overline{V}\right) \qquad \sum \overline{F} = \frac{\partial}{\partial t}\int_V \rho \overline{V} dV + \int_A \rho \overline{V}\overline{V} \cdot \overline{n} dA$$

3. **Energiayhtälö**

$$P = me,\ p = e = u + \frac{v^2}{2} + gz$$

Q + W = ΔE

$$\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{\partial}{\partial t}\int_V \rho\, e dV + \int_A \rho\left(e + \frac{p}{\rho}\right)\overline{V} \cdot \overline{n} dA - \frac{dW_\mu}{dt}$$

$W = W_s + W_p + W_\mu$

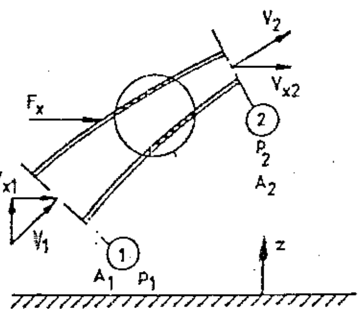
1 - D VIRTAUKSEN YHTÄLÖT

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ Jatkuvuusyhtälö

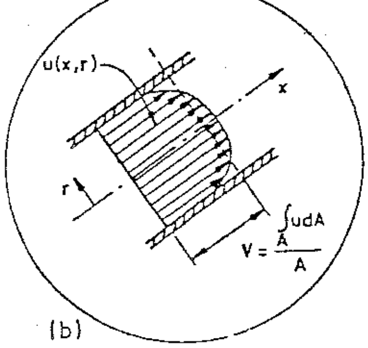
$F_x = \dot{m}\left(\alpha_2 V_{x2} - \alpha_1 V_{x1}\right)$ Liikeyhtälö

$F_y = \dot{m}\left(\alpha_2 V_{y2} - \alpha_1 V_{y1}\right)$

$$\frac{w_s}{g} + \frac{p_1}{\rho g} + \beta_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \beta_2 \frac{V_2^2}{2g} + z_2 + h_v$$ Energiayhtälö



(a)



(b)

$$\frac{p}{\rho} + gz + \frac{u^2}{2} = \text{vakio} \qquad \text{Bernoulli}$$

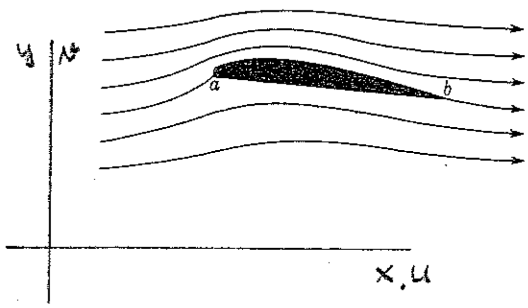
$$\int \frac{dp}{\rho} + gz + \frac{u^2}{2} = \text{vakio (kokoonpuristuva)}$$

KITKATON, KOKOONPURISTUMATON VIRTAUS (2-dim)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{f_x}{\rho} \qquad (2)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{f_y}{\rho} \qquad (3)$$



POTENTIAALIVIRTAUS

– pyörteetön virtaus $\qquad \overline{\omega} = \nabla x \overline{V} = 0$

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \rightarrow \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \qquad \text{(2 - D virtaus)}$$

toteutuu, jos

$$u = \frac{\partial \phi}{\partial x}, v = \frac{\partial \phi}{\partial y} \qquad (4)$$

$$\nabla^2 \phi = 0 \qquad (4 \rightarrow 1)$$

$$\frac{p}{\rho} + gh + \frac{V^2}{2} = \text{vakio} \qquad (4 \rightarrow 2)$$

– virtafunktio

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \qquad (\nabla \cdot \overline{V} \text{ toteutuu})$$

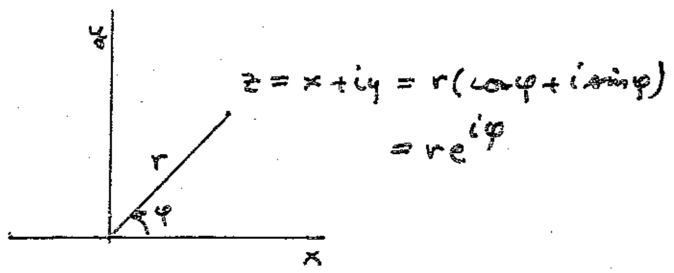
$$\nabla^2 \psi = 0, \quad \text{jos} \quad \zeta = 0$$

$$w(z) = \phi + i\psi \qquad z = x + iy = r(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$

$$\frac{dw}{dz} = u - iv$$

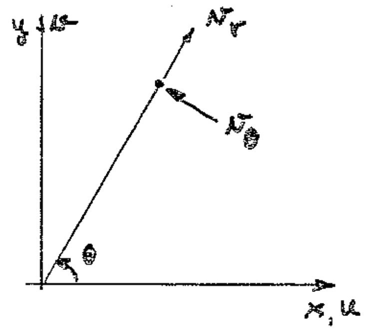
$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y}$$



$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r}$$

$$v_\theta = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$



$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad \nabla \cdot \overline{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \qquad \nabla \cdot \overline{V} = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}$$

$$w(z) = \phi + i\psi$$

$w = Uze^{-i\alpha}$ tasainen nopeusjakautuma

$w = m \ln z$ lähde

$w = -\frac{\mu}{z}$ kaksoislähde

$w = -i\frac{\Gamma}{2\pi}\ln z$ kiertovirtaus

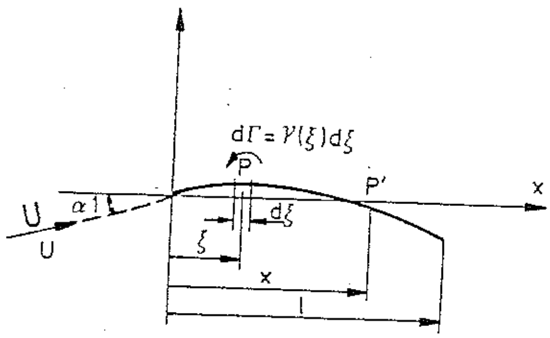
Ohuen siiven teoria

$$\frac{dy}{dx} = \tan\beta = \frac{U\sin\alpha + v'}{U\cos\alpha + u'} \approx \alpha + \frac{v'}{U}$$

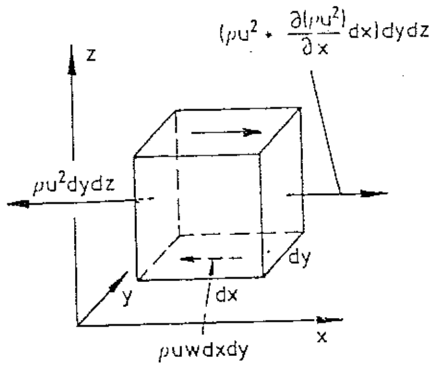
$$v' = U\left(\frac{dy}{dx} - \alpha\right)$$

$$v' = \frac{1}{2\pi}\int_o^l \frac{\gamma(\xi)}{x-\xi}d\xi$$

$$U\left(\frac{dy}{dx} - \alpha\right) = \frac{1}{2\pi}\int_o^l \frac{\gamma(\xi)}{x-\xi}d\xi$$



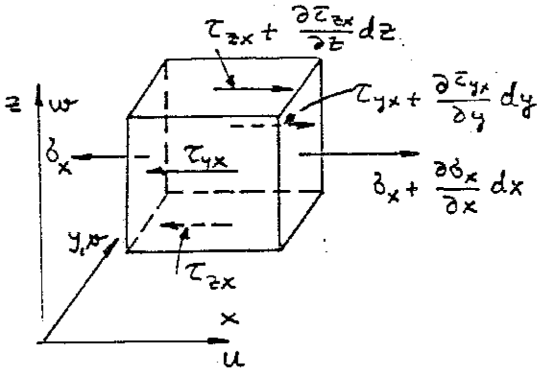
LIIKEYHTÄLÖT (Navier-Stokesin yhtälöt)



$$\overline{F} = \frac{\partial}{\partial t}\int_V \rho\, \overline{V} dV + \int_A \rho\, \overline{V}\overline{V} \cdot \overline{n} dA$$

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2)}{\partial x} + \frac{\partial (\rho uv)}{\partial y} + \frac{\partial (\rho uw)}{\partial z} = F_x$$

$$F_x = \left[f_x + \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right]$$



$$\sigma_x = 2\eta \left(\frac{\partial u}{\partial x} - \frac{1}{3} \nabla \cdot \overline{V} \right) - p$$

$$\tau_{xy} = \eta \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

$$\tau_{xz} = \eta \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \qquad \text{jne.}$$

$$\rho \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \right) = f_x - \frac{\partial p}{\partial x} + \eta \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

$$\rho \left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \right) = f_y - \frac{\partial p}{\partial y} + \eta \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

$$\rho \left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \right) = f_z - \frac{\partial p}{\partial z} + \eta \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = f_i - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2}$$

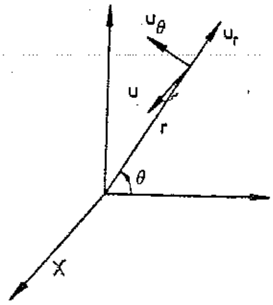
$$\frac{\partial \rho}{\partial t} + \frac{\partial (u_i \rho)}{\partial x_i} = 0$$

$$\frac{D\overline{V}}{Dt} = \bar{f} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \overline{V}$$

$$\nabla \cdot \overline{V} = 0$$

$$\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u}{\partial x}=0 \qquad (\nabla\cdot\overline{V}=0)$$

$$\rho\left[\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u\frac{\partial u_r}{\partial x}-\frac{u_\theta^2}{r}\right]=-\frac{\partial p}{\partial r}$$



$$+\eta\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}+\frac{\partial^2 u_r}{\partial x^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right]+f_r$$

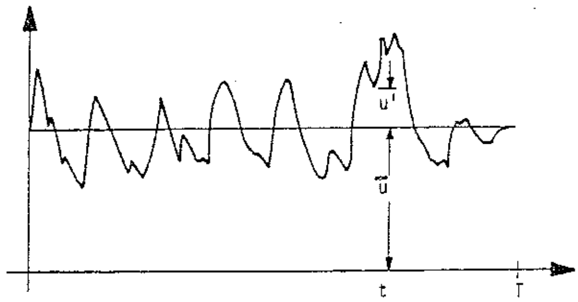
$$\rho\left[\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_r u_\theta}{r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u\frac{\partial u_\theta}{\partial x}\right]=-\frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$+\eta\left[\frac{\partial^2 u_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{\partial^2 u_\theta}{\partial x^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right]+f_\theta$$

$$\rho\left[\frac{\partial u}{\partial t}+u_r\frac{\partial u}{\partial r}+\frac{u_\theta}{r}\frac{\partial u}{\partial \theta}+u\frac{\partial u}{\partial x}\right]=-\frac{\partial p}{\partial x}$$

$$+\eta\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{\partial^2 u}{\partial x^2}\right]+f_x$$

TURBULENTTI VIRTAUS



- virtaus on aina 3-dim.
- Navier-Stokesin yhtälöt voimassa

$u=\overline{u}+u'$

$v=\overline{v}+v'$

$p=\overline{p}+p' \qquad \overline{\phi}=\frac{1}{\Delta t}\int_t^{t+\Delta t}\phi\, dt$

- sijoitus Navier-Stokesin yhtälöihin ja integrointi

$$\rho\left(\overline{u}\frac{\partial \overline{u}}{\partial x}+\overline{v}\frac{\partial \overline{u}}{\partial y}\right)=-\frac{\partial \overline{p}}{\partial x}+\frac{\partial}{\partial x}\left(\eta\frac{\partial \overline{u}}{\partial x}-\rho\,\overline{u'^2}\right)+\frac{\partial}{\partial y}\left(\eta\frac{\partial \overline{u}}{\partial y}-\rho\,\overline{u'v'}\right)$$

$$\rho\left(\overline{u}\frac{\partial \overline{v}}{\partial x}+\overline{v}\frac{\partial \overline{v}}{\partial y}\right)=-\frac{\partial \overline{p}}{\partial y}+\frac{\partial}{\partial x}\left(\eta\frac{\partial \overline{v}}{\partial x}-\rho\,\overline{u'v'}\right)+\frac{\partial}{\partial y}\left(\eta\frac{\partial \overline{v}}{\partial y}-\rho\,\overline{v'^2}\right)$$

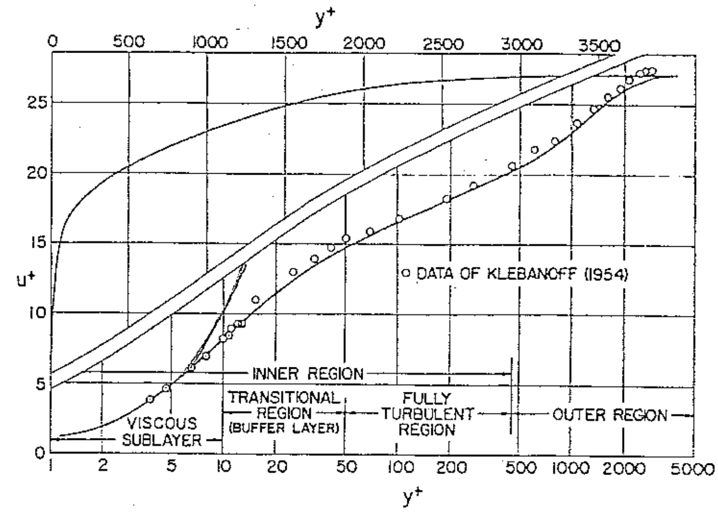
$$\frac{\partial \overline{u}}{\partial x}+\frac{\partial \overline{v}}{\partial y}=0$$

$$T_u = \frac{\sqrt{\overline{u'^2}}}{\overline{u}}$$

$$k = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$$

$$\tau = \tau_l + \tau_t = \rho\,\nu\,\frac{\partial \overline{u}}{\partial y} - \rho\,\overline{u'v'}$$

$$\tau = \rho\left(\nu + \nu_t\right)\frac{\partial \overline{u}}{\partial y}$$



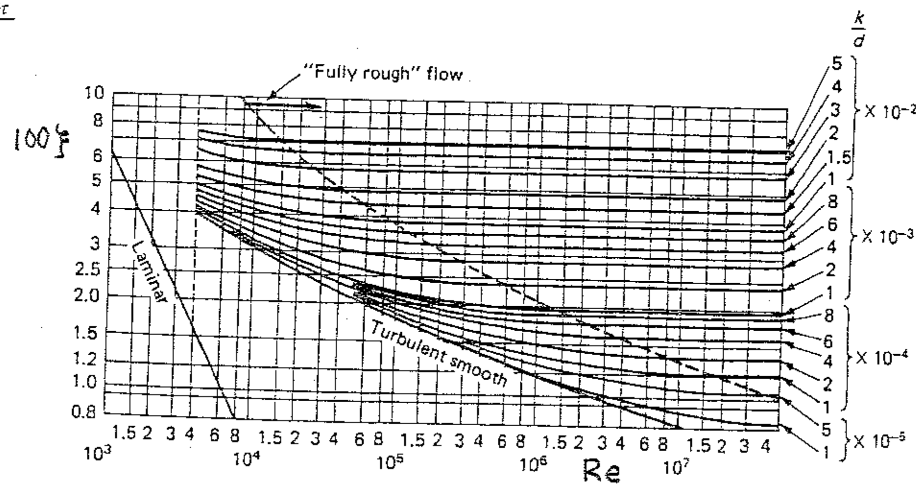
$$\tau_t = -\rho\,\overline{u'v'} = \rho\,\nu_t\frac{\partial u}{\partial y}$$

$$\tau = \tau_s \quad \text{ja} \quad \nu_t = l^2\left(\frac{\partial u}{\partial y}\right), \qquad l = ky$$

$$u^+ = \frac{1}{k}\ln y^+ + c \qquad\qquad k = 0{,}4,\ c = 5{,}5$$

$$u^+ = y^+$$

$$u^+ = \frac{u}{u_\tau}, \qquad u_\tau = \sqrt{\frac{\tau_s}{\rho}} \qquad y^+ = \frac{y u_\tau}{\nu}$$



$$f = \frac{\tau_s}{\frac{1}{2}\rho V^2}$$

$$\xi = 4f$$

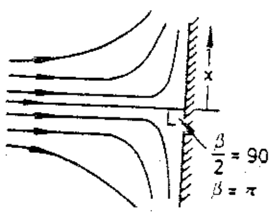
$$\Delta p = \xi\,\rho\,\frac{V^2}{2}\frac{l}{d_h}\ , \quad d_h = \frac{4A}{P}$$

RAJAKERROSYHTÄLÖT

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

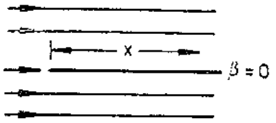
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

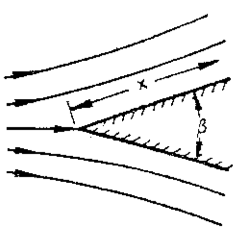
$$u_\infty\frac{du_\infty}{dx} = -\frac{1}{\rho}\frac{dp}{dx}$$



$$u_\infty = c x^m$$

$$m = \frac{\beta/\pi}{2 - \beta/\pi}$$





$$\psi = \sqrt{\nu\, x c x^{m}}\, f(\eta)$$

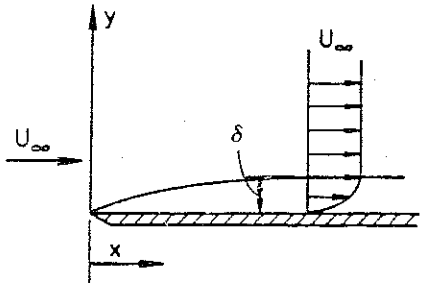
$$\eta = y\sqrt{\frac{cx^{m}}{\nu x}} \qquad\qquad f''' + \left(\frac{m+1}{2}\right) ff'' + m\left(1-(f')^{2}\right) = 0$$

β	m	f''(0)
2π	∞	∞
π	1	1,233 patopiste
1,57	0,333	0,759
0,627	0,111	0,510
0	0	0,332 tasolevy
-0,314	-0,0476	0,222
-0,624	-0,091	0 virtaus irtoaa

$$f_x = \frac{\tau_s(x)}{\frac{1}{2}\rho\, u_\infty^{2}} = 2 f''(0)\,\mathrm{Re}_x^{-1/2}$$

$$f = \frac{1}{x}\int_0^x f_x\, dx$$

TASOLEVY. LAMINAARI RAJAKERROS



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^{2} u}{\partial y^{2}}$$

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

$$\psi = \sqrt{\nu\, x u_\infty}\, f(\eta) \qquad\qquad u = U_\infty f'$$

$$\eta = y\sqrt{\frac{u_\infty}{\nu x}} \qquad\qquad v = -\frac{1}{2}\left(\frac{\nu u_\infty}{x}\right)^{1/2}\left(f - \eta f'\right)$$

$$ff'' + 2f''' = 0 \qquad\qquad f(o) = f'(o) = 0,\; f'(\infty) = 1$$

$$f = \sum_{n=0}^{\infty}\frac{an}{n!}\eta^{n} \qquad\qquad f(o) = f'(o)$$

$\eta = y\sqrt{\frac{u_\infty}{\nu x}}$	$f(\eta)$	$f'(\eta) = \frac{u}{u_\infty}$	$f''(\eta)$
0	0	0	0,33206
0,4	0,02656	0,13277	0,33147
0,8	0,10611	0,26471	0,32739
1,2	0,23795	0,39378	0,31659
1,6	0,42032	0,51676	0,29667
2,0	0,65003	0,62977	0,26675
2,4	0,92230	0,72809	0,22809
2,8	1,23099	0,81152	0,18401
3,2	1,56911	0,87609	0,13913
3,6	1,92954	0,92333	0,09809
4,0	2,30576	0,95552	0,06424
4,4	2,69238	0,97587	0,03896
4,8	3,08534	0,98779	0,02187
5,2	3,48189	0,99425	0,01134
5,6	3,88031	0,99748	0,00543
6,0	4,27964	0,99898	0,00240
6,4	4,67938	0,99961	0,00098
6,8	5,07928	0,99987	0,00037
7,2	5,47925	0,99996	0,00013
7,6	5,87924	0,99999	0,00004
8,0	6,27923	1,00000	0,00001
8,4	6,67923	1,00000	0,00000

INTEGRAALIMENETELMÄ

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\frac{1}{\rho}\frac{\partial \tau}{\partial y} \qquad \tau=\rho\left(v+v_t\right)\frac{\partial u}{\partial y}$$

$$\frac{d}{dx}\int_o^{\delta} u\left(u_\infty-u\right)dy+\frac{du_\infty}{dx}\int_o^{\delta}\left(u_\infty-u\right)dy=\frac{\tau_s}{\rho}$$

$$\delta^*=\int_o^{\delta}\left(1-\frac{u}{u_\infty}\right)dy \qquad \theta=\int_o^{\delta}\frac{u}{u_\infty}\left(1-\frac{u}{u_\infty}\right)dy$$

$$\frac{d\left(\theta\, u_\infty^2\right)}{dx}+\delta^* u_\infty\frac{du_\infty}{dx}=\frac{\tau_s}{\rho}$$

$$H=\frac{\delta^*}{\theta}$$

$$\frac{d\theta}{dx}=\frac{\tau_s}{\rho\, u_\infty^2}-(H+2)\frac{\theta}{u_\infty}\frac{du_\infty}{dx}$$

$$\theta^2=\frac{0{,}45v}{u_\infty^6}\int_o^x u_\infty^5 dx \qquad \text{Thwaites}$$

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– integraalimenetelmä

$$\frac{u}{u_\infty}=\left(\frac{y}{\delta}\right)^{1/7}$$

$$\tau_s=0{,}0225\rho\, u_\infty^2\left(\frac{\delta\, u_\infty}{v}\right)^{-1/4} \qquad \text{(turbulentti putkivirtaus)}$$

$$\delta=0{,}37x\,\mathrm{Re}_x^{-1/5} \qquad \text{(käytetään putkivirtauksen tuloksia integraalimenetelmässä)}$$

$$f_x=0{,}0576\,\mathrm{Re}_x^{-1/5}, \qquad f=\frac{1}{l}\int_o^l f_x dx$$